MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

63rd Annual

# AMC 12 B

**American Mathematics Contest 12 B**

**Wednesday, February 22, 2012**

## INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 12 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

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The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

*Students who score 100 or above or finish in the top 5% on this AMC 12 will be invited to take the 30th annual American Invitational Mathematics Examination (AIME) on Thursday, March 15, 2012 or Wednesday, March 28, 2012. More details about the AIME and other information are on the back page of this test booklet.*

2012

# AMC 12 B

## DO NOT OPEN UNTIL WEDNESDAY, FEBRUARY 22, 2012

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. PLEASE READ THE MANUAL BEFORE FEBRUARY 22, 2012. Nothing is needed from inside this package until February 22.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 12 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, e-mail, internet or media of any type is a violation of the competition rules.*


**The American Mathematics Competitions**

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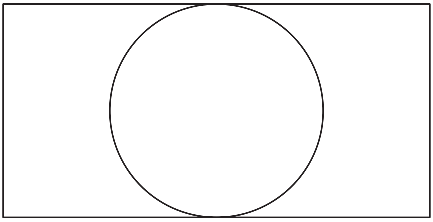
1. Each third-grade classroom at Pearl Creek Elementary has 18 students and 2 pet rabbits. How many more students than rabbits are there in all 4 of the third-grade classrooms?

   **(A)** 48 **(B)** 56 **(C)** 64 **(D)** 72 **(E)** 80

2. A circle of radius 5 is inscribed in a rectangle as shown. The ratio of the length of the rectangle to its width is 2 : 1. What is the area of the rectangle?



   **(A)** 50 **(B)** 100 **(C)** 125 **(D)** 150 **(E)** 200

3. For a science project, Sammy observed a chipmunk and a squirrel stashing acorns in holes. The chipmunk hid 3 acorns in each of the holes it dug. The squirrel hid 4 acorns in each of the holes it dug. They each hid the same number of acorns, although the squirrel needed 4 fewer holes. How many acorns did the chipmunk hide?

   **(A)** 30 **(B)** 36 **(C)** 42 **(D)** 48 **(E)** 54

4. Suppose that the euro is worth 1.30 dollars. If Diana has 500 dollars and Étienne has 400 euros, by what percent is the value of Étienne's money greater than the value of Diana's money?

   **(A)** 2 **(B)** 4 **(C)** 6.5 **(D)** 8 **(E)** 13

5. Two integers have a sum of 26. When two more integers are added to the first two integers the sum is 41. Finally when two more integers are added to the sum of the previous four integers the sum is 57. What is the minimum number of even integers among the 6 integers?

   **(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

6. In order to estimate the value of $x - y$ where $x$ and $y$ are real numbers with $x > y > 0$, Xiaoli rounded $x$ up by a small amount, rounded $y$ down by the same amount, and then subtracted her rounded values. Which of the following statements is necessarily correct?

   **(A)** Her estimate is larger than $x - y$.

   **(B)** Her estimate is smaller than $x - y$.

   **(C)** Her estimate equals $x - y$.

   **(D)** Her estimate equals $y - x$.

   **(E)** Her estimate is 0.

7. Small lights are hung on a string 6 inches apart in the order red, red, green, green, green, red, red, green, green, green, and so on continuing this pattern of 2 red lights followed by 3 green lights. How many feet separate the 3rd red light and the 21st red light?

   **Note:** 1 foot is equal to 12 inches.

   **(A)** 18 **(B)** 18.5 **(C)** 20 **(D)** 20.5 **(E)** 22.5

8. A dessert chef prepares the dessert for every day of a week starting with Sunday. The dessert each day is either cake, pie, ice cream, or pudding. The same dessert may not be served two days in a row. There must be cake on Friday because of a birthday. How many different dessert menus for the week are possible?

   **(A)** 729 **(B)** 972 **(C)** 1024 **(D)** 2187 **(E)** 2304

9. It takes Clea 60 seconds to walk down an escalator when it is not operating, and only 24 seconds to walk down the escalator when it is operating. How many seconds does it take Clea to ride down the operating escalator when she just stands on it?

   **(A)** 36 **(B)** 40 **(C)** 42 **(D)** 48 **(E)** 52

10. What is the area of the polygon whose vertices are the points of intersection of the curves $x^2 + y^2 = 25$ and $(x-4)^2 + 9y^2 = 81$ ?

    **(A)** 24 **(B)** 27 **(C)** 36 **(D)** 37.5 **(E)** 42

11. In the equation below, $A$ and $B$ are consecutive positive integers, and $A$, $B$, and $A + B$ represent number bases:

$$132_A + 43_B = 69_{A+B}.$$

What is $A + B$?

**(A)** 9 **(B)** 11 **(C)** 13 **(D)** 15 **(E)** 17

12. How many sequences of zeros and/or ones of length 20 have all the zeros consecutive, or all the ones consecutive, or both?

**(A)** 190 **(B)** 192 **(C)** 211 **(D)** 380 **(E)** 382

13. Two parabolas have equations $y = x^2 + ax + b$ and $y = x^2 + cx + d$, where $a$, $b$, $c$, and $d$ are integers (not necessarily different), each chosen independently by rolling a fair six-sided die. What is the probability that the parabolas have at least one point in common?

**(A)** $\frac{1}{2}$ **(B)** $\frac{25}{36}$ **(C)** $\frac{5}{6}$ **(D)** $\frac{31}{36}$ **(E)** 1

14. Bernardo and Silvia play the following game. An integer between 0 and 999, inclusive, is selected and given to Bernardo. Whenever Bernardo receives a number, he doubles it and passes the result to Silvia. Whenever Silvia receives a number, she adds 50 to it and passes the result to Bernardo. The winner is the last person who produces a number less than 1000. Let $N$ be the smallest initial number that results in a win for Bernardo. What is the sum of the digits of $N$?

**(A)** 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

15. Jesse cuts a circular paper disk of radius 12 along two radii to form two sectors, the smaller having a central angle of 120 degrees. He makes two circular cones, using each sector to form the lateral surface of a cone. What is the ratio of the volume of the smaller cone to that of the larger?

**(A)** $\frac{1}{8}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{\sqrt{10}}{10}$ **(D)** $\frac{\sqrt{5}}{6}$ **(E)** $\frac{\sqrt{10}}{5}$

16. Amy, Beth, and Jo listen to four different songs and discuss which ones they like. No song is liked by all three. Furthermore, for each of the three pairs of the girls, there is at least one song liked by those two girls but disliked by the third. In how many different ways is this possible?

**(A)** 108 **(B)** 132 **(C)** 671 **(D)** 846 **(E)** 1105

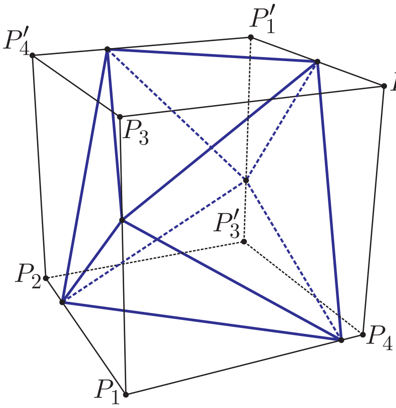
17. Square $PQRS$ lies in the first quadrant. Points $(3,0)$, $(5,0)$, $(7,0)$, and $(13,0)$ lie on lines $SP$, $RQ$, $PQ$, and $SR$, respectively. What is the sum of the coordinates of the center of the square $PQRS$?

**(A)** 6 **(B)** 6.2 **(C)** 6.4 **(D)** 6.6 **(E)** 6.8

18. Let $(a_1, a_2, \dots, a_{10})$ be a list of the first 10 positive integers such that for each $2 \le i \le 10$ either $a_i + 1$ or $a_i - 1$ or both appear somewhere before $a_i$ in the list. How many such lists are there?

**(A)** 120 **(B)** 512 **(C)** 1024 **(D)** 181,440 **(E)** 362,880

19. A unit cube has vertices $P_1$, $P_2$, $P_3$, $P_4$, $P_1'$, $P_2'$, $P_3'$, and $P_4'$. Vertices $P_2$, $P_3$, and $P_4$ are adjacent to $P_1$, and for $1 \le i \le 4$, vertices $P_i$ and $P_i'$ are opposite to each other. A regular octahedron has one vertex in each of the segments $P_1P_2$, $P_1P_3$, $P_1P_4$, $P_1'P_2'$, $P_1'P_3'$, and $P_1'P_4'$. What is the octahedron's side length?



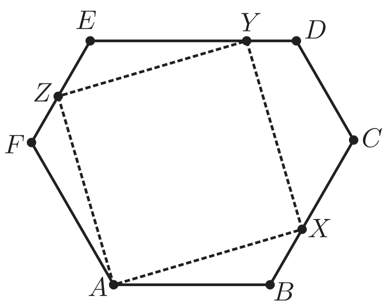

**(A)** $\frac{3\sqrt{2}}{4}$ **(B)** $\frac{7\sqrt{6}}{16}$ **(C)** $\frac{\sqrt{5}}{2}$ **(D)** $\frac{2\sqrt{3}}{3}$ **(E)** $\frac{\sqrt{6}}{2}$

20. A trapezoid has side lengths 3, 5, 7, and 11. The sum of all the possible areas of the trapezoid can be written in the form of $r_1\sqrt{n_1}+r_2\sqrt{n_2}+r_3$, where $r_1, r_2$, and $r_3$ are rational numbers and $n_1$ and $n_2$ are positive integers not divisible by the square of a prime. What is the greatest integer less than or equal to

$$r_1 + r_2 + r_3 + n_1 + n_2\,?$$

**(A)** 57 **(B)** 59 **(C)** 61 **(D)** 63 **(E)** 65

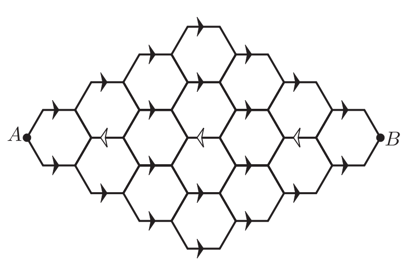
21. Square $AXYZ$ is inscribed in equiangular hexagon $ABCDEF$ with $X$ on $\overline{BC}$, $Y$ on $\overline{DE}$, and $Z$ on $\overline{EF}$. Suppose that $AB = 40$ and $EF = 41(\sqrt{3}-1)$. What is the side-length of the square?




**(A)** $29\sqrt{3}$ **(B)** $\frac{21}{2}\sqrt{2}+\frac{41}{2}\sqrt{3}$ **(C)** $20\sqrt{3}+16$ **(D)** $20\sqrt{2}+13\sqrt{3}$

**(E)** $21\sqrt{6}$

22. A bug travels from $A$ to $B$ along the segments in the hexagonal lattice pictured below. The segments marked with an arrow can be traveled only in the direction of the arrow, and the bug never travels the same segment more than once. How many different paths are there?




**(A)** 2112 **(B)** 2304 **(C)** 2368 **(D)** 2384 **(E)** 2400

23. Consider all polynomials of a complex variable, $P(z) = 4z^4 + az^3 + bz^2 + cz + d$, where $a, b, c,$ and $d$ are integers, $0 \le d \le c \le b \le a \le 4$, and the polynomial has a zero $z_0$ with $|z_0| = 1$. What is the sum of all values $P(1)$ over all the polynomials with these properties?

**(A)** 84 **(B)** 92 **(C)** 100 **(D)** 108 **(E)** 120

24. Define the function $f_1$ on the positive integers by setting $f_1(1) = 1$ and if $n = p_1^{e_1} p_2^{e_2} \cdots p_k^{e_k}$ is the prime factorization of $n > 1$, then

$$f_1(n) = (p_1 + 1)^{e_1 - 1}(p_2 + 1)^{e_2 - 1} \cdots (p_k + 1)^{e_k - 1}.$$

For every $m \ge 2$, let $f_m(n) = f_1(f_{m-1}(n))$. For how many $N$ in the range $1 \le N \le 400$ is the sequence $(f_1(N), f_2(N), f_3(N), \ldots)$ unbounded?

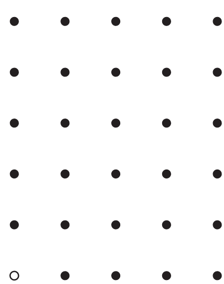
**Note:** a sequence of positive numbers is unbounded if for every integer $B$, there is a member of the sequence greater than $B$.

**(A)** 15 **(B)** 16 **(C)** 17 **(D)** 18 **(E)** 19

25. Let $S = \{(x, y) : x \in \{0, 1, 2, 3, 4\}, y \in \{0, 1, 2, 3, 4, 5\}, \text{ and } (x, y) \ne (0, 0)\}$. Let $T$ be the set of all right triangles whose vertices are in $S$. For every right triangle $t = \triangle ABC$ with vertices $A$, $B$, and $C$ in counter-clockwise order and right angle at $A$, let $f(t) = \tan(\angle CBA)$. What is

$$\prod_{t \in T} f(t)\,?$$

$S$



**(A)** 1 **(B)** $\frac{625}{144}$ **(C)** $\frac{125}{24}$ **(D)** 6 **(E)** $\frac{625}{24}$

MATHEMATICAL ASSOCIATION OF AMERICA

## American Mathematics Competitions

### WRITE TO US!

*Correspondence about the problems and solutions for this AMC 12*
*and orders for publications should be addressed to:*

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*The problems and solutions for this AMC 12 were prepared by the MAA's Committee on the AMC 10 and AMC 12 under the direction of AMC 12 Subcommittee Chair:*

Prof. Bernardo M. Abrego

### 2012 AIME

The 30th annual AIME will be held on Thursday, March 15, with the alternate on Wednesday, March 28. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above or finish in the top 2.5% of the AMC 10, or if you score 100 or above or finish in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the 41st Annual USA Mathematical Olympiad (USAMO) on April 24-25, 2012. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

## PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site: amc.maa.org